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A MODEL FOR STATIONARY SLUGS

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Abstract—Studies of stationary slugs were carried out in order to obtain an understanding of the slug regime that exists for gas/liquid flow in a pipeline. A simple model is developed which considers the front to be a hydraulic jump. The liquid height in the tail decreases in two stages: an inviscid rapidly changing flow; and a slowly changing viscous flow. For a fixed liquid height in front of the stationary slug the first stage of the tail is a Benjamin bubble, for which the volumetric flow of the liquid is a minimum value. These results provide support for theoretical arguments used previously to develop necessary conditions for the existence of a slug regime.

Key Words: slug flow, stationary slugs, gas/liquid flow, horizontal flow

INTRODUCTION

A stationary hydraulic jump can be formed by introducing a disturbance into a fluid flowing under a gate located in a pipe that is inclined downward. The jump occurs downstream of the gate. If air is allowed to bleed into the space between the jump and the gate, the jump will reach a stationary position and the rate of entrainment of air by the jump will be equal to that in the downstream collector, the liquid behind the jump will have a gradually sloping tail and will not fill the whole length of pipe behind the jump. The static jump that is formed resembles the rapidly moving slugs observed, under certain conditions, when gas and liquid flow in a pipeline and is, therefore, called a stationary slug.

Kennison (1933) discussed the use of a hydraulic jump to remove air from conduits. Kalinske & Robertson (1943) explored this application in a systematic study in which they showed how stationary slugs can be formed and in which they provided measurements of pressure drop and rate of air entrainment.

The possibility of using the stationary slug as a means to obtain a better understanding of the hydrodynamics of moving slugs has recently been pointed out by Jepson (1987a, b), who presented measurements of velocity profiles and of void distributions.

The present paper develops a simple model for a stationary slug and tests it by determining the pressure drop, the shape of the tail and the necessary conditions for existence. Of particular importance is the analysis of a tail as a composite of a rapidly changing inviscid flow and a slowly changing viscous flow.

The motivation comes from a recent paper by Ruder *et al.* (1989), which developed necessary conditions for the existence of moving slugs, observed in gas liquid flows. They represented the tail of a slug by an inviscid Benjamin bubble (Benjamin 1968) which sheds liquid at a volumetric rate, given as

$$V = 0.542 \frac{\pi D^2}{4} \sqrt{gD}, \qquad [1]$$

where D is the pipe diameter and g is the acceleration due to gravity. Some direct support for this analysis is obtained from an observation by Wallis & Dobson (1973) that Benjamin bubbles form directly behind slugs. By using conservation of mass, [1] was employed to define a minimum liquid height in front of the slug, h_{L1}, below which slugs cannot exist. By using the notion that the front of the slug is a hydraulic jump, a minimum Froude number for slugs to exist was defined. A goal of this paper is to explore the accuracy of this model through studies with the much simpler stationary slug.

Experiments with stationary slugs have been conducted in this laboratory by Johnson (1987), Burban (1988) and Merrick (1989). The results in this paper were obtained in equipment that used a modified version of designs developed by them. Water flowed in pipes with i.d. = 5.09 and 2.54 cm, that were inclined downward at an angle of 5°. The liquid discharged into a separator. The pipe was long enough for the liquid layer flowing into the separator to have reached an equilibrium height, where the gravitational pull on the liquid is balanced by the resisting stress at the wall.

Pressure measurements confirm that the front of the stationary slug is a hydraulic jump. Photographic measurements are used to support the model for the tail that is developed. For a fixed h_{L1} (controlled by the gate opening), the minimum liquid volumetric flow for a stationary slug is found to agree with [1]. For liquid flows larger than this, the flow immediately behind the slug cannot be described as a Benjamin bubble, even though an inviscid approximation is still valid. A maximum V for a fixed h_{L1} and a maximum h_{L1} for a fixed V are defined from the properties of a hydraulic jump.

THEORY

(A) Outline of a physical model for a stationary slug

A sketch of a stationary slug is given in figure 1. The pattern has three parts. The front of the slug, between stations 1 and 3, is pictured to be a hydraulic jump. The strong vortex formed in the jump entrains air which is broken into small bubbles. Some of these bubbles are recycled back into air space; others are carried by the liquid into the body of the slug between stations 3 and 4. For simplicity, the flow in the slug body is considered to be uniform. At station 3, bubbles are distributed over the whole section except for a small region close to the bottom of the pipe. As the bubbles are carried downstream they rise upward and many are at the top of the pipe when they exit at the tail. The increase in velocity in the slug tail occurs in two stages. In the front part, the liquid height drops rather suddenly; a rapid acceleration of the liquid occurs so that inertia forces dominate and an inviscid approximation can be made. The downstream part of the tail is characterized by a gradual change in height, so that viscous effects associated with the resistance of the wall become important.

This model for a stationary slug is quite similar to the model for a moving slug used by Ruder *et al.* (1989). It is anticipated that many of the results obtained in this paper could be applicable to a moving slug, when viewed in a coordinate system moving with the slug. The liquid velocity, u_{L1}, is analogous to the difference in the slug velocity (which is close to the gas velocity) and the velocity of the slow liquid layer in front of a moving slug, $C - u_{L1}$.

(B) The hydraulic jump

If the resistance of the wall is ignored the following equation is derived by writing a momentum balance between stations 1 and 3, with the assumption of plug flow and a uniform distribution of voids at station 3:

$$P_1A_3 + \rho_L u_{L1}^2 A_{L1} + \rho_G u_{G1}^2 (A_3 - A_{L1}) + \rho_G g(A_3 - A_{L1})h_{pc2}\cos\beta + \rho_L g A_{L1} h_{pc1}\cos\beta + \rho_{13} g L_{13} A_3 \sin\beta$$
$$= P_3A_3 + \rho_L u_{L3}^2 A_3(1-\epsilon_3) + \rho_G u_{G3}^2 A_3\epsilon_3 + \rho_G \epsilon_3 g A_3 h_{pc3}\cos\beta + \rho_L(1-\epsilon_3) g A_3 h_{pc3}\cos\beta, \quad [2]$$

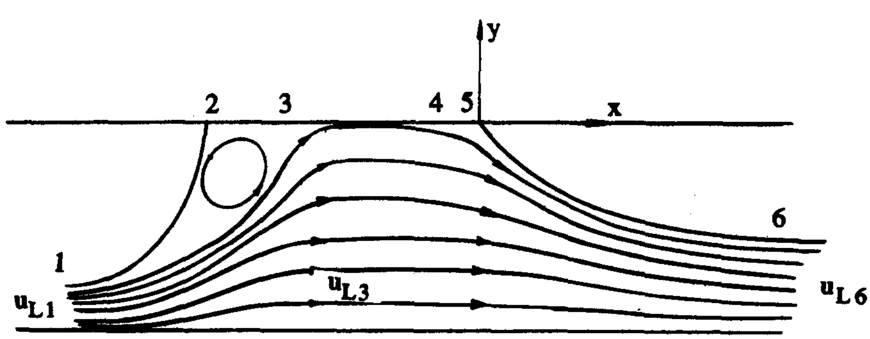


Figure 1. Physical model of the stationary drag.

where P_1 and P_3 are pressures at the top of the pipe, ρ_L and ρ_G are the densities of liquid and air, ϵ_3 is the void fraction at station 3, A_{L1} is the area of the liquid at station 1, A_3 is the pipe area, β is the angle of pipe inclination, L_{13} is the length from station 1 to station 3, h_{pc1}, h_{pc2} and h_{pc3} are the centroid heights for hydrostatic pressure in the liquid layer, the air pocket and the slug body respectively, ρ_{13} is the mixture density between stations and 1 and 3:

$$\rho_{13} = [\rho_L(1 - \epsilon_{13}) + \rho_G \epsilon_{13}]; \quad [3]$$

ϵ_{13} is the average void fraction between stations 1 and 3.

Conservation of mass for the liquid and the gas gives

$$u_{L1} A_{L1} = u_{L3} A_3 (1 - \epsilon_3) \quad [4]$$

and

$$u_{G1}(A_3 - A_{L1})u = u_{G3} A_3 \epsilon_3 = R_A, \quad [5]$$

where R_A is the net rate at which air is entrained by the slug. The following relation for the pressure increase from the air pocket to station 3 due to the hydraulic jump is obtained by substituting [4] and [5] into [2]:

$$\frac{(\Delta P)_h}{\rho_L g} = \frac{1}{g}\left[\frac{A_{L1}}{A_3} u_{L1}^2 - (1 - \epsilon_3) u_{L3}^2\right] + \left[\frac{A_{L1}}{A_3} h_{pc1} - (1 - \epsilon_3) h_{pc3}\right] \cos\beta$$

$$+ \frac{\rho_G}{\rho_L g}\left[\left(1 - \frac{A_{L1}}{A_3}\right) u_{G1}^2 - \epsilon_3 u_{G3}^2\right] + \frac{\rho_G}{\rho_L}\left[h_{pc2}\left(1 - \frac{A_{L1}}{A_3}\right) - \epsilon_3 h_{pc3}\right] \cos\beta. \quad [6]$$

In this equation, the pressure drop associated with the force of gravity in the flow direction, $\rho_{13} g L_{13} \sin\beta$, is not included because the location of station 3 is not known exactly. This pressure drop is included as an independent item in the calculation of total pressure drop (see section E). For an air/water system, the last two terms in [6] are very small compared to the first two, so they can be neglected.

(C) Flow characteristics of an unaerated slug tail

The presence of voids makes an exact analysis of the tail difficult. These bubbles are not uniformly distributed at the slug tail; they disrupt the interface where they erupt from the upper part of the tail; they are associated with irreversible effects. Consequently, an analysis was carried out by ignoring the presence of bubbles. The results are expected to represent a limiting behavior as the gas fraction approaches zero. Surprisingly, it is found that this simplified analysis represents observations with large voids.

The gas space behind the slug is at constant pressure. For an inviscid flow the velocity of liquid increases from station 5 to station 6 along the free streamline because of the loss of potential energy associated with gravity. Ruder *et al.* (1989) ignored the influence of voids and used the model of a gravity current presented by Benjamin (1968), whereby station 5 is a stagnation point and the interface makes an angle $\pi/3$ with the top wall. They assumed this described a limiting behavior of a slug and called the tail, under these circumstances, a Benjamin bubble.

For flow in a two-dimensional channel, the final height of the liquid downstream predicted by this theory is

$$h_{L6} = 0.5B, \quad [7]$$

where B is the height of the channel. The downstream velocity of the liquid is

$$u_{L6} = \sqrt{gB} \quad [8]$$

and the volumetric flow of liquid shed by the tail is given by

$$V = 0.5B\sqrt{gB}W, \quad [9]$$

where W is the width of the channel. For a pipe flow,

$$h_{L6} = 0.563D, \quad [10]$$

$$u_{L6} = 0.935\sqrt{gD} \quad [11]$$

and

$$V = 0542\frac{\pi D^2}{4}\sqrt{gD}. \quad [12]$$

These results are supported in extensive experiments by Zukowski (1966), who showed that the inclination angle of the pipeline has no influence on the motion of bubbles if it is $<3°$ and that the bubble velocity does change with the viscosity if the Reynolds number is large enough.

The experiments with stationary slugs described in this paper show that [12] defines a lower liquid volumetric flow for which a slug could be formed. However, stationary slugs were also observed at larger V. If the velocity profile at station 4 is assumed to be approximately uniform, the only way for the tail of the slug to behave as an inviscid flow under these conditions is for the starting point of the slug tail not to be a stagnation point. This could be justified on physical grounds if it is realized that the air emerges out of the tail at station 5 and that a thin film of air can exist at the top of the slug (see figure 11).

The velocity at station 6 is now given as

$$u_{L6}^2 = u_0^2 + 2g(B - h_{L6})\cos\beta + gL_{56}\sin\beta \quad [13]$$

for a rectangular channel, where u_0 is the effective velocity at station 5, L_{56} is the length from station 5 to station 6. Another relation for u_{L6} can be obtained by applying a mass and momentum balance between stations 4 and 6:

$$u_{L4}B = u_{L6}h_{L6} \quad [14]$$

and

$$P_4B + \tfrac{1}{2}\rho_L gB^2\cos\beta - \tfrac{1}{2}\rho_L gh_{L6}^2\cos\beta = \rho_L u_{L6}^2 h_{L6} - \rho_L u_{L4}^2 B - \rho_L gL_{45}B\sin\beta - \rho_L g(1-\epsilon_{56})L_{56}B\sin\beta. \quad [15]$$

Here, the pressure in the gas space behind the slug has been taken as zero, L_{45} is the length from station 4 to station 5 and ϵ_{56} is the average void fraction between stations 5 and 6 in the slug tail.

By applying Bernoulli's equation along the streamline from station 4 to station 5, the following is obtained:

$$P_4 = \tfrac{1}{2}\rho_L u_0^2 - \tfrac{1}{2}\rho_L u_{L4}^2 - \rho_L gL_{45}\sin\beta. \quad [16]$$

In the experiments, $\beta = 5°$ and $\cos\beta \approx 1$. If $1 - \epsilon_{56}$ is approximated by h_{L6}/B, the following relation for u_{L6} is obtained by combining [14]–[16]:

$$u_{L6} = \frac{[g(B^2 - h_{L6}^2) + Bu_0^2]B}{(2B - h_{L6})h_{L6}} + gL_{56}\sin\beta\frac{2B}{2B - h_{L6}}. \quad [17]$$

Equation [13] is substituted into [17] to eliminate u_{L6}^2, so that

$$u_0^2 + 2g(B - h_{L6}) = \frac{[g(B^2 - h_{L6}^2) + Bu_0^2]B}{(2B - h_{L6})h_{L6}} + gL_{56}\sin\beta\frac{h_{L6}}{2B - h_{L6}}. \quad [18]$$

Because $\sin\beta[h_{L6}/(2B - h_{L6})] \ll 1$, the last term in [18] can be neglected and a solution for h_{L6} is obtained:

$$h_{L6} = \frac{1}{2}\left(B + \frac{u_0^2}{g}\right). \quad [19]$$

By substituting [19] into [13] and neglecting the last term in [13] (because the length of the first stage of the slug tail, L_{56}, is $< B$ and $\sin\beta$ is very small), the following relation for u_{L6} is obtained:

$$u_{L6} = \sqrt{gB}. \quad [20]$$

From [19] and [20] the volumetric flow is given as

$$V = 0.5\sqrt{gB}\left(B + \frac{u_0^2}{g}\right)W. \qquad [21]$$

It is noted that h_{L6}, V and u_{L6} are the same as for a Benjamin bubble if $u_0 = 0$. For $u_0 \neq 0$ the volumetric flow and the height at station 6 increase, while the velocity remains the same. The Froude number at station 6 is given as

$$\mathrm{Fr}_6 = \frac{u_{L6}}{\sqrt{gh_{L6}}} = \frac{\sqrt{B}}{\left[0.5\left(B + \frac{u_0^2}{g}\right)\right]^{0.5}} \qquad [22]$$

and the Froude number inside the slug is

$$\mathrm{Fr}_4 = \frac{u_{L4}}{\sqrt{gB}} = 0.5 + 0.5\frac{u_0^2}{gB}. \qquad [23]$$

For $u_0 = 0$, $h_{L6} = 0.5B$, $\mathrm{Fr}_6 = \sqrt{2}$ and $\mathrm{Fr}_4 = 0.5$, the liquid flow is supercritical in the slug tail and subcritical inside the slug. The maximum value of the effective velocity u_0 would be $\sqrt{gB}$. For this case, $h_{L6} = B$, $\mathrm{Fr}_6 = 1$ and $\mathrm{Fr}_4 = 1$; the liquid fills the whole channel. Normally, $0 < u_0 < \sqrt{gB}$, $0.5B < h_{L6} < B$, $1 < \mathrm{Fr}_6 < \sqrt{2}$ and $0.5 < \mathrm{Fr}_4 < 1.0$.

By carrying out the same analysis for a circular pipe, the following relation can be obtained when the influences of void fraction and inclination are neglected, as was done for a rectangular channel:

$$\xi^2(1 - \cos\epsilon) + \eta\cos\epsilon - (\tfrac{2}{3}\pi)\sin^3\epsilon + \xi^2\left(\frac{u_0^2}{gD}\right) = 0, \qquad [24]$$

in which

$$\xi = \frac{\epsilon - 0.5\sin 2\epsilon}{\pi} \qquad [25]$$

and

$$\epsilon = \arccos\left[2\left(\frac{h}{D} - 0.5\right)\right]. \qquad [26]$$

Such an equation can be solved only by a numerical method. When $u_0 = 0$, the Benjamin bubble solution is obtained.

(D) Shape of the slug tail

The shape of the tail in the region from station 5 to station 6, sketched in figure 1, can be calculated for a rectangular channel, with methods outlined by Benjamin (1968), by neglecting the influence of inclination and void fraction. As shown by von Karman (Yih 1965), the angle between the top of the channel and the free surface is $\pi/3$. This inviscid solution gives a free surface that levels out to a constant value of $h_{l6} = B/2$ downstream. For a situation in which station 5 is not a stagnation point, the h_{L6} value calculated from [18] is larger than $B/2$.

Farther downstream the flow has to adjust to the outlet condition, where $h_L < h_{L6}$, The change of height in this region is gradual so that shallow water theory can be used to simplify the momentum balance and the wall stress can be approximated by a pseudo-steady state assumption.

For a two-dimensional channel, the momentum and mass balances are as follows (Dressler 1949; Miya *et al.* 1971):

$$u\frac{\partial u}{\partial x} = -\frac{\tau_w}{\rho_L h} + g\cos\beta - g\sin\beta\frac{\partial h}{\partial x} \qquad [27]$$

and

$$Whu = V, \qquad [28]$$

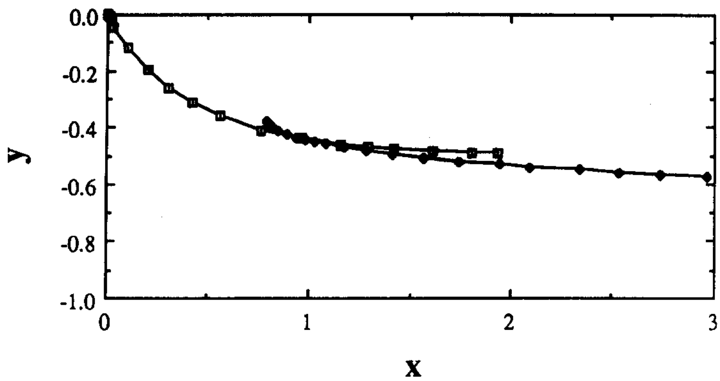


Figure 2. Slug tail match (channel).

where τ_w is the average resisting stress at the wall. If u is eliminated between [27] and [28], the following equation for the height of the free surface is obtained:

$$\frac{\mathrm{d}h}{\mathrm{d}x}=\frac{-\dfrac{\tau_w}{\rho_L h}+g\cos\beta}{g\sin\beta-\dfrac{V^2}{h^3W^2}}. \quad [29]$$

A solution of [29] , using the Blasius equation to approximate τ_w, is presented in figure 2 as the filled points. As shown in figure 1, y is the perpendicular distance from the top wall of the channel, made dimensionless with the channel height, and x is the dimensionless distance along the channel starting from station 5. There is a region where neither the inviscid nor the shallow water assumption are valid. For simplicity, this is ignored and the two limiting solutions outlined above are simply matched at a location where they give approximately the same slope.

For a circular pipe the flow is three-dimensional; a solution by conformal mapping is not, strictly, applicable. An approximate solution is obtained by treating the flow at the symmetry plane as if it were two-dimensional. Variables are made dimensionless using the diameter of the pipe and the velocity u_{L4}. From [10] and [11] the dimensionless height and velocity at station 6 are given as 0.563 and 1.725, respectively. The influence of pipe inclination and void fraction is neglected, as was done in the analysis of a channel flow. From Bernoulli's equation

$$u_{L6}^2=-2gy_6, \quad [30]$$

where $y_6=h_{L6}-1$. In order for [30] to give the desired values of u_{L6} and h_{L6}, dimensionless g must be given a value of 3.402. The interfacial shape is calculated by the same method used by Benjamin for a rectangular channel, except that "a" is changed from 1.49 to 1.418 (because along the free surface the equation describing dimensionless $q^2=u^2+v^2$ changes from $-8y$ to $-6.805y$) and the function

$$\zeta=\frac{aZ+2bZ^2}{1+cZ+bZ^2} \quad [31]$$

changes to

$$\zeta=\frac{aZ+1.725bZ^2}{1+cZ+bZ^2} \quad [32]$$

(because the dimensionless downstream velocity is now 1.725 rather than 2). The interfacial shape far downstream is, again, calculated using shallow water theory and the pseudo-steady state assumption.

The open points in figure 3 are the inviscid solution for the front part of the tail and the filled points are the viscous solution for the back part. Again, matching is done at a point where the interfacial slopes for the two solutions are approximately equal.

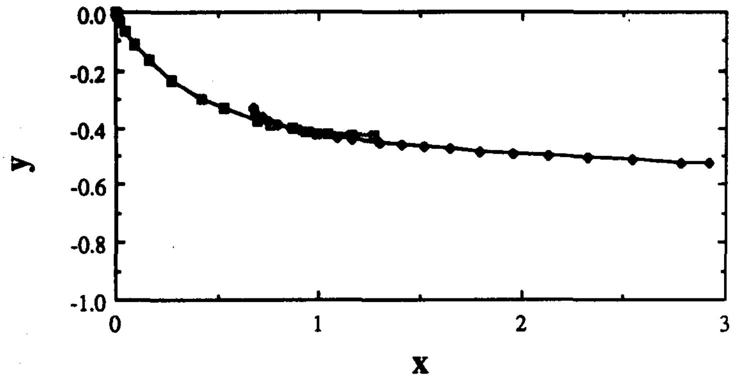


Figure 3. Slug tail match (pipe).

The important result displayed in figures 2 and 3 is that the matching occurs where h_{L6} is very close to the values given by [7] and [10]. This means that the procedure used by Ruder *et al.* (1989) of using only the inviscid solution to establish necessary conditions is reasonable.

(E) Pressure drop in a pipe

The pressure drop over the stationary slug in a pipe is considered as being composed of four components: the pressure change associated with the hydraulic jump in the front, ΔP_h; a frictional pressure drop in the body of the slug, ΔP_f; the pressure rise associated with the velocity change from station 4 to station 5 at the slug tail, ΔP_r; and the pressure drop to support the weight of the slug due to pipe inclination, ΔP_g. Thus the total pressure change is given as

$$\Delta P_T = \Delta P_h + \Delta P_r + \Delta P_g - \Delta P_f. \qquad [33]$$

The pressure drop associated with the hydraulic jump is given by [6]. It is usually much larger than ΔP_f and ΔP_r. The pressure drop in the body of the slug is pictured as mainly resulting from the force of gravity due to pipe inclination, ΔP_g, and the frictional drag at the wall, ΔP_f. The former is

$$\Delta P_g = g[\rho_L(1-\epsilon_3) + \rho_G\epsilon_3]L_S \sin\beta, \qquad [34]$$

where L_S is the slug length (from station 2 to station 5).

The pressure drop due to the frictional drag at the pipe wall is

$$\Delta P_f = \frac{\tau_w \pi D L}{A_3}. \qquad [35]$$

When calculating τ_w, there is a question as to what is the effective density. Dukler & Hubbard (1975) suggested a mixture of density

$$\rho = \rho_L(1-\epsilon_3) + \rho_G\epsilon_3. \qquad [36]$$

Other researchers (Singh & Griffiths 1970; Bonnecaze *et al.* 1971) suggested that $\rho = \rho_L$. In this research, ρ_L is used. The wall stress is then approximated with the Blasius equation as follows:

$$\tau_w = 0.046\left(\frac{D}{\nu_L}\right)^{-0.20} \frac{\rho_L}{2} u_{L3}^{1.8}, \qquad [37]$$

where ν_L is the kinematic viscosity of the liquid. For stationary slugs, ΔP_f is found to be small enough to be neglected.

The pressure drop ΔP_r is approximated by applying Bernoulli's equation along the streamline between stations 4 and 5 at the top of the pipe, so that

$$\Delta P_r = P_5 - P_4 = \tfrac{1}{2}\rho_L u_{L3}^2 - \tfrac{1}{2}\rho_L u_0^2. \qquad [38]$$

In fact, the gas and liquid flow at the top of the pipe are very chaotic and the void fraction could be very large. Surprisingly, it is found when the liquid density, ρ_L, is used in Bernoulli's equation that ΔP_r calculated from [38] is very close to that measured in experiments. For $u_o = 0$, the tail of the stationary slug is a Benjamin bubble. If it is assumed that the volumetric flow out of the tail is given by [4], then, from conservation of mass,

$$u_{L3} = \frac{0.542\sqrt{gD}}{(1-\epsilon_3)} \qquad [39]$$

and

$$\Delta P_r = \tfrac{1}{2}\rho_L u_{L3}^2 = \frac{0.147\rho_L gD}{(1-\epsilon_3)^2}. \qquad [40]$$

For $u_0 \neq 0$, ΔP_r will be less than given by [40].

(F) Necessary conditions for the existence of an unaerated stationary slug

It has been shown that, if the influence of void fraction is neglected, the volumetric flow out of the tail is a minimum when the slug tail can be described by a Benjamin bubble. For a channel, this minimum is given by [9] and it equals the volumetric flow in the liquid layer in front of the stationary slug, so that

$$\frac{h_{L1}u_{L1}}{B\sqrt{gB}} = 0.5. \qquad [41]$$

The existence of a slug is also limited by the requirement that the energy dissipation is positive, as suggested by Ruder *et al.* (1989); this leads to the condition

$$\frac{u_{L1}}{\sqrt{gB}} > 1. \qquad [42]$$

Equations [41] and [42] are represented by curves 1 and 3 in figure 4. It should be noted that these two lines intercept at $h_{L1}/B = 0.5$. At this condition both the front and the tail of the slug are Benjamin bubbles. Curves 1 and 3 define the necessary conditions for the existence of a stationary slug in a channel. Similar results were also obtained for a pipe flow by Ruder *et al.* (1989)—these are represented by curves 1 and 2 in figure 5.

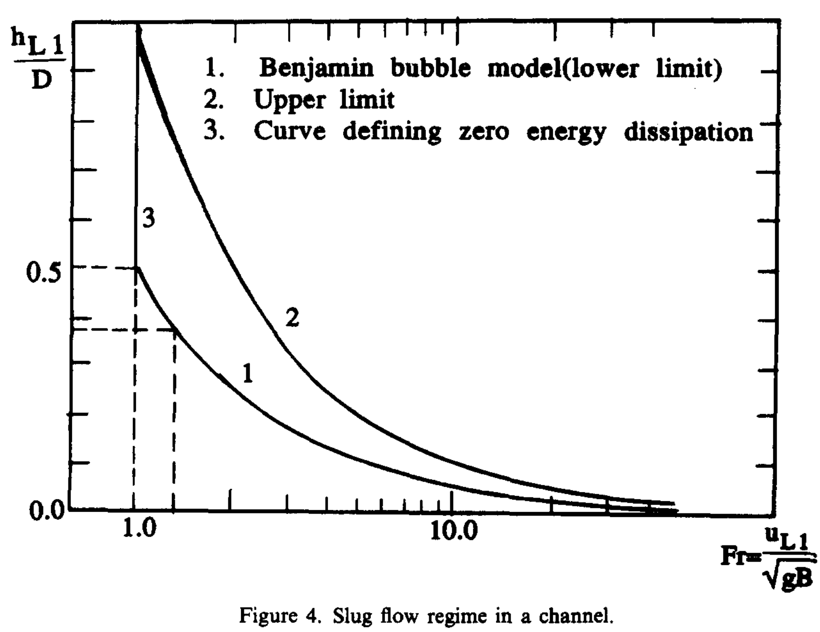


Figure 4. Slug flow regime in a channel.

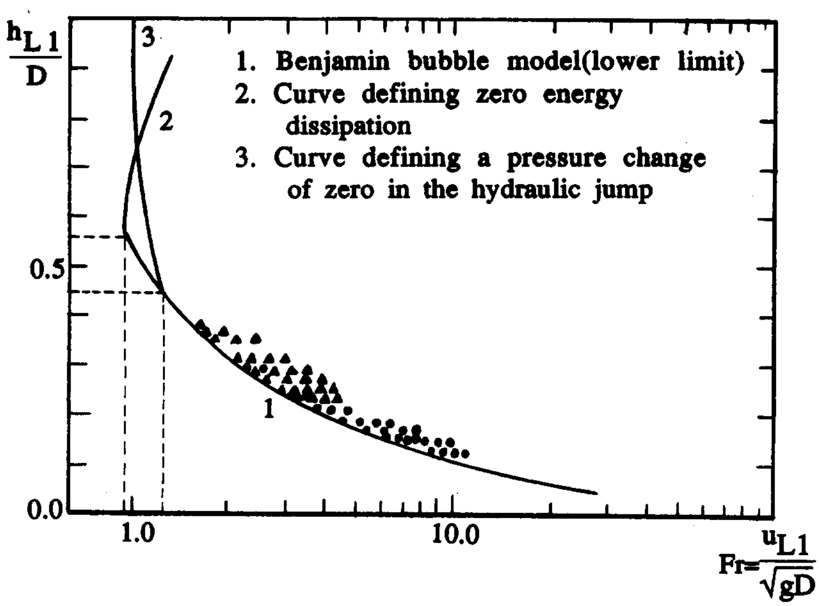


Figure 5. Slug flow regime in circular pipes: ▲, 5.09 cm i.d. pipe; ●, 2.50 cm i.d. pipe.

Differences in the flow characteristics of stationary slugs in a channel and a pipe are emphasized when an upper limit for the liquid flow is explored. For a two-dimensional channel, the analysis in section C reveals that $u_{L4}/\sqrt{gB} = 1$ requires that the liquid fill the whole channel at the tail. This leads to the limiting condition

$$\frac{u_{L4}}{\sqrt{gB}} > 1, \qquad [43]$$

which is represented by curve 2 in figure 4. The same result can be obtained by applying the conservation equations between stations 4 and 6 and requiring that the dissipation of mechanical energy is positive or zero.

A condition similar to [43] cannot be obtained for a stationary slug in a pipe, either from [24] or from the requirement that energy dissipation is positive. For example, the last condition requires that

$$\frac{u_{L4}^2}{gD} < \frac{1 - 2\dfrac{h_{L6}}{d} + 2\dfrac{A_{L6}h_{pc6}}{A_{L4}D}}{\left(\dfrac{A_{L4}}{A_{L6}} - 1\right)^6}. \qquad [44]$$

As h_{L6} approaches D, the denominator in [44] approaches zero much faster than the numerator, so u_{L4}^2/gD does not appear to have an upper limit if aeration is ignored.

For a rectangular channel, a Froude number with exact physical meaning is used to describe the hydraulic jump, as in section C. Several different definitions of Froude number have been used for a flow in a pipe (Benjamin 1968; Kouba & Jepson 1989). The Froude number characterizing the liquid flow in this research is based on the pipe diameter,

$$\mathrm{Fr} = \frac{u_{L1}}{\sqrt{gD}}, \qquad [45]$$

so that the results presented can be compared with the study of moving slugs presented by Ruder *et al.* (1989).

Ruder & Hanratty (1990) explored the possibility that curve 2 in figure 5 would represent a transition to plug flow for gas/liquid flow in a pipeline. They found that this is, indeed, a necessary condition for the existence of slugs but that the actual breakdown of a slug occurs at slightly higher Froude numbers. At $\mathrm{Fr} = (C - u_{L1})/\sqrt{gD} = 1.2$, the gas space was found to resemble a symmetric bubble and at Fr = 2 the hydraulic jump in the front slug took the appearance of a staircase. Here C is the velocity of the slug and u_{L1} is the velocity of the liquid in the carpet in front of the slug. The term $(C - u_{L1})$ is the velocity of the liquid seen by an observer moving with the slug. It is analogous to the u_{L1} defined for a stationary slug.

Observations of stationary slugs also show that the hydraulic jump at the front of the slug is initiated at higher Froude numbers than given by curve 2 in figure 5. Experiments performed for the conditions represented by curve 1 in figure 5 reveal that with decreasing $\mathrm{Fr} = u_{L1}/\sqrt{gD}$, the pressure increase associated with the hydraulic jump decreases and that the degree of aeration decreases. At a small enough Froude numbers, aeration ceases and, at Froude numbers slightly below this value, the pressure change in the front of the slug becomes zero. This condition of zero pressure increase was explored as a stablilty condition for a hydraulic jump. The physical notion behind this is that the vortical motion behind the jump should be associated with an unfavorable pressure gradient along the top wall.

In a two-dimensional channel the pressure drop over a hydraulic jump is given by

$$\frac{(\Delta P)_h}{\rho_L g} = \frac{1}{g}\left(\frac{h_{L1}}{B}u_{L1}^2 - u_{L3}^2\right) + \left(\frac{h_{L1}}{B}\frac{h_{L1}}{2} - \frac{B}{2}\right) \qquad [46]$$

if there is no aeration and the influence of inclination is neglected. The mass balance is

$$u_{L1}h_{L1} = u_{L3}B \qquad [47]$$

and, if the tail is a Benjamin bubble,

$$u_{L3} = 0.5\sqrt{gB}. \qquad [48]$$

If [47] and [48] are substituted into [46], the following conditions are obtained for $(\Delta P)_h = 0$:

$$\left(\frac{h_{L1}}{B}\right) = 0.366 \qquad [49]$$

and

$$\frac{u_{L1}}{\sqrt{gB}} = 1.366. \qquad [50]$$

In a similar way, conditions for $(\Delta P)_h = 0$ for a pipe, can be developed:

$$\frac{h_{L1}}{D} = 0.4456 \qquad [51]$$

and

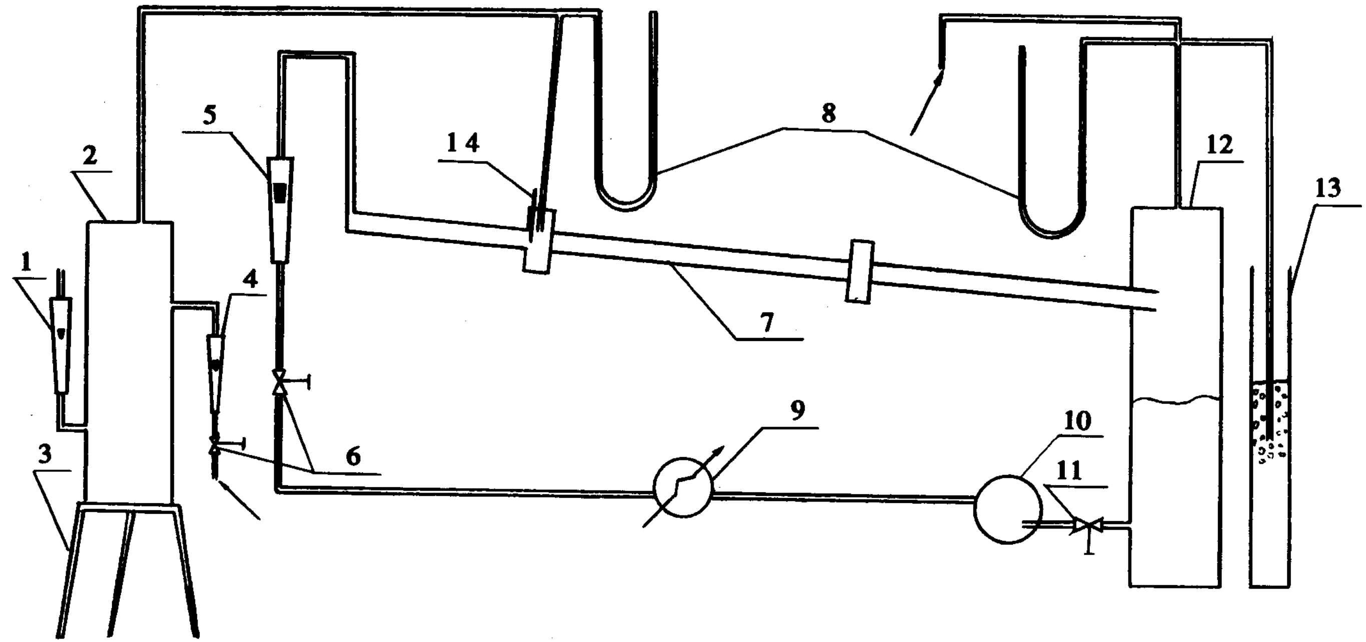
$$\frac{u_{L1}}{\sqrt{gD}} = 1.2579. \qquad [52]$$

It is noted that [52] is independent of pipe diameter and corresponds closely to the condition where Ruder & Hanratty (1990) observed the formation of a symmetric gas bubble.

For h_{L1}/D larger than that required for the tail to be a Benjamin bubble, the condition $\Delta P_h = 0$ requires a slightly smaller $u_{L1}/\sqrt{gD}$ than given by [52]. This is represented by curve 3 in figure 5. If the criterion $(\Delta P)_h > 0$ is correct, then slugs could exist only to the right of curves 2 and 3 in figure 5.

EXPERIMENTS

Experiments were carried out in Plexiglas pipes, with i.d. = 5.09 and 2.50 cm, which were inclined downward to make it possible to create stationary slugs. If a horizontal pipe is used a hydraulic jump can be formed, but the liquid fills the cross section downstream of the jump over the whole length of the pipe. During these experiments, the inclination angle was chosen as 5°. The flow loop is shown in figure 6. The liquid flow rate was measured by the rotameter 5. At the inlet of the test section, the pipe was linked to air tank 2 to keep the pressure at the front of the slug at a constant



1. Rotameter, 2. Air Tank, 3. Base, 4. Rotameter, 5. Rotameter
6. Valve, 7. Testing section, 8. Manometer, 9. Heat Exchanger,
10. Pump, 11. Valve, 12. Liquid Storage Tank,
13. Pressure Adjuster 14. Gate

Figure 6. The flow system.

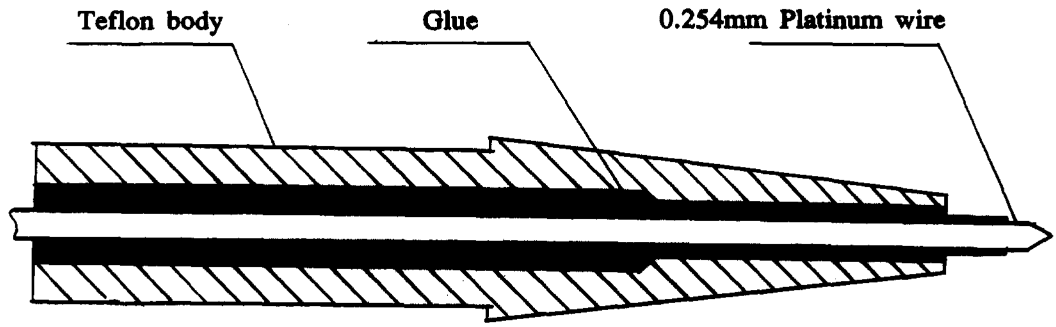


Figure 7. The probe used in voidage measurements.

value during the experiments and to supply the slug with the air which is needed for aeration. A sliding gate was inserted into the pipe to control the height of the liquid layer in the front of the slug.

In order to control the air pressure behind the slug, air was supplied to the liquid storage tank. The only exit for the air was through a tube partially submerged into another water-filled tank that is called a pressure adjuster. By changing the height of the tube, the pressure in the back of the slug can be adjusted.

After water was supplied to the pipe and the part of the pipe before the test section filled up with water, valve 6 was quickly opened to produce a rapid increase in water flow rate. In this way, a slug was created in the pipe. When the slug has just been created, the liquid flow rate is usually larger than the minimum value given by [1]; the tail is not a Benjamin bubble. Then, the liquid flow rate is carefully decreased. When the liquid flow rate is lower than a certain value, the slug will suddenly collapse.

The height of the liquid layer in front of the slug was measured by a contact probe. The pressure distribution along the slug was measured both at the top and at the bottom of the slug. When taking

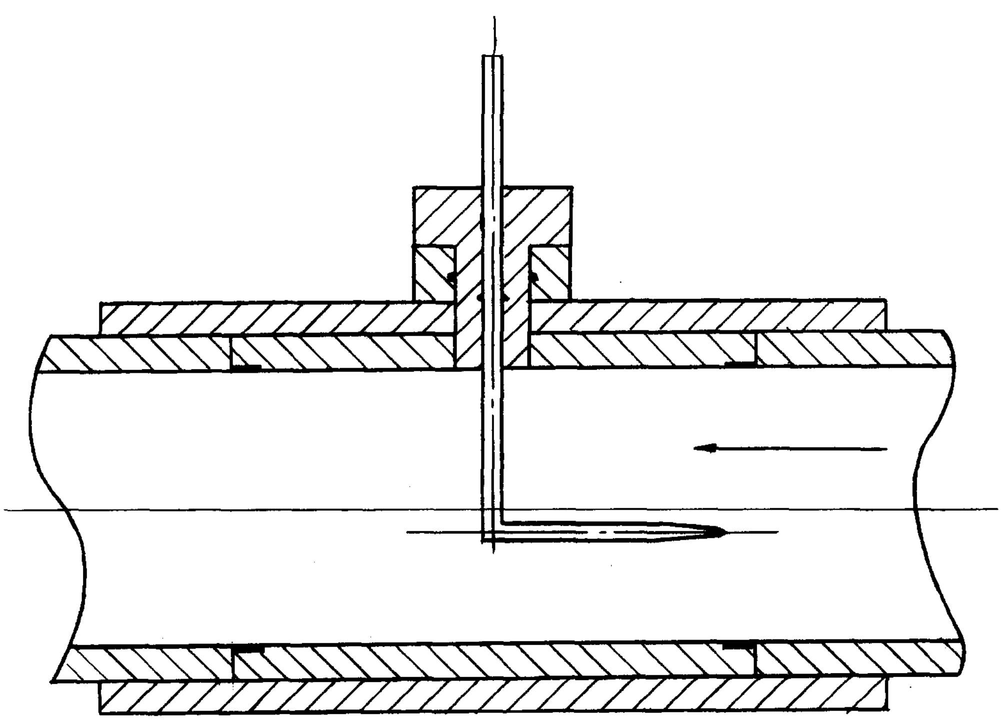


Figure 8. The voidage test section.

photographs of the slug, a Plexiglas box filled with water covered the outside of the pipe. In this way, distortion due to the difference in the refractive indices of water and Plexiglas was greatly decreased.

An electric-resistance method was used to determine the void fraction of air inside slugs (Herringe & Davis 1974; Serizawa *et al.* 1975; Thang & Davis 1979). The probe used in this experiment is shown in figure 7, and the setup for the measurement in figure 8. The probe is inserted into the slugs. There are two brass rings before and behind the probe. The probe and the brass rings were connected to a frequency counter and a square-wave generator to form an electric circuit. When a bubble is not on the probe the square-wave signal produced by the generator can go to the frequency counter through the probe, water and brass rings. When a bubble arrives at the probe, the electric circuit is broken. By counting the fraction of the time the circuit is broken, the local void fraction inside the slug can be obtained. The average void fraction was obtained by integrating the local void fraction over the cross section of the pipe.

RESULTS

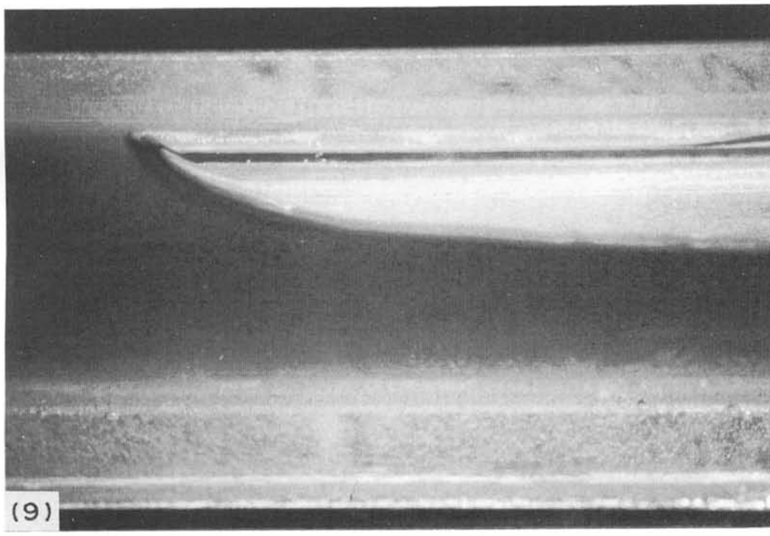
(A) Slug tail shape

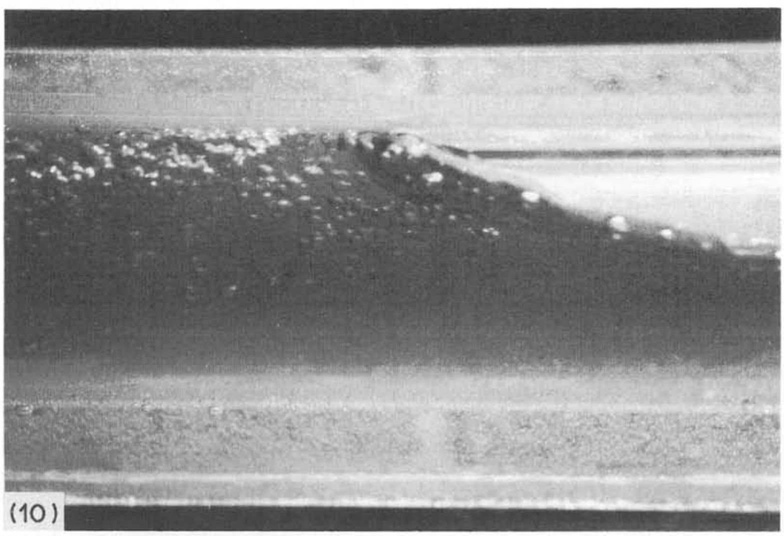
Figure 9 is a photograph of a slug tail when the liquid flow rate was kept at the shedding rate for a Benjamin bubble and $\mathrm{Fr} \approx 1.7$. Under this flow condition, there is no aeration in the slug. Figures 9 and 3 are in almost perfect agreement, which means that the front of the slug tail can be represented as a Benjamin bubble.

However, in most situations, there is always some aeration in the slug due to the hydraulic jump in the front of the slug. Figure 10 is a photograph of a slug tail taken when $\mathrm{Fr} \approx 2.5$. The liquid flow rate was equal to the shedding rate for a Benjamin bubble and the void fraction inside the slug was about 0.13. Under such a flow condition, unsteady waves occur at the slug tail; bubbles flow with the liquid out the back of the slug, especially at the top. However, the tail still keeps the basic characteristics of a Benjamin bubble. The angle between the free surface of the liquid and the top wall of the pipe is nearly equal to $\pi/3$ and the height of the free surface decreases, on average, at the same rate as a Benjamin bubble. Even though a large amount of bubbles collected, the liquid still touched the top of the pipe. Figure 11 is a photograph of a slug tail when $\mathrm{Fr} = 2.5$ and the liquid flow rate is 12% larger than the shedding rate for a Benjamin bubble. Under these conditions, the void fraction is the same as for the experiment in figure 10. However, the shape of the slug tail is different from that of a Benjamin bubble, especially at the beginning of the tail. A slipping regime exists in which a thin layer of air is formed between the top wall of the pipe and the liquid. The liquid departs very gradually from the top wall of the pipe, initially, and then drops much more quickly. The final height of the tail is also larger than for the slug shown in figure 10. Slugs for which the tail is not a Benjamin bubble were obtained by increasing the liquid flow rate while keeping the height of liquid layer in front of the slug constant. This caused an increase in the liquid velocities u_{L1} and u_{L3}. The increase in u_{L1} is much larger than the increase in u_{L3}. From [6], it can be seen that the pressure increase associated with the hydraulic jump increases and the pressure increase at the rear, ΔP_r, decreases because of the occurrence of a relative velocity, u_0 (see [38]). This is associated with a larger flow out at the tail.

(B) Pressure drop over the stationary slug

Figure 12 presents measurements of the pressure distribution along a slug, both at the top and at the bottom when the liquid flow rate is kept at the shedding rate for a Benjamin bubble and $\mathrm{Fr} = 2.7$. As is indicated, there are two sharp increases in the pressure at the top of the slug. The one in front of the slug is associated with the hydraulic jump; the one at the rear is interpreted as being associated with a flow stagnation at the beginning of the slug tail. When the height of the liquid film at station 1 was decreased and the liquid flow rate was kept at the shedding rate for a Benjamin bubble, the Froude number increased and the aeration became stronger. Many bubbles concentrate on the top of the slug, especially at the rear. Table 1 lists the average void fraction measured in the experiments, the measured pressure drop at the rear of the slug and the pressure drop calculated from [40], when the tail of the slug is a Benjamin bubble. When $\mathrm{Fr} > 2.5$





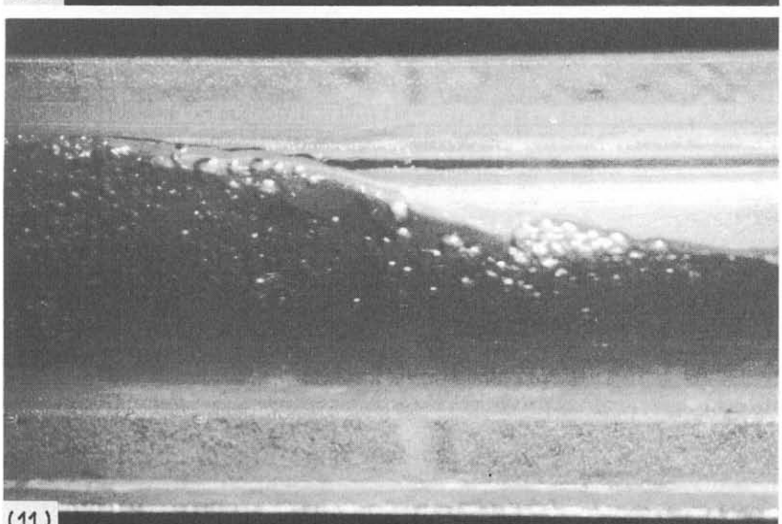


Figure 9. An unaerated slug tail (Fr = 1.7).

Figure 10. A strong aerated slug tail (Fr = 2.5) at the minimum volumetric flow.

Figure 11. A non-Benjamin bubble slug tail at a liquid flow that is 12% larger than the minimum.

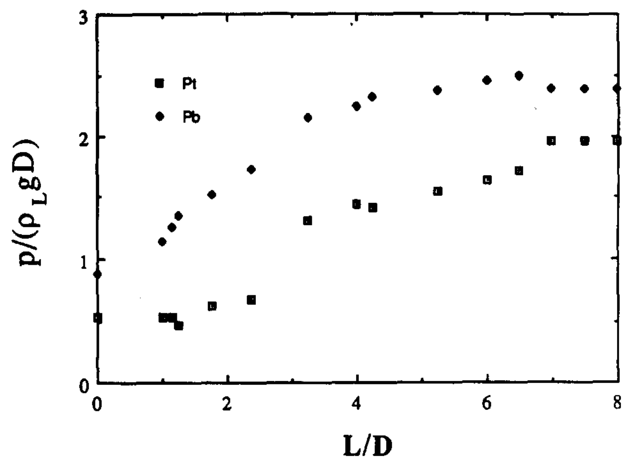


Figure 12. Pressure distribution along the slug (Fr = 2.7).

the aeration becomes strong and the void fraction becomes large, but the measured pressure increase at the rear of the slug is still close to that calculated from [40]. This supports the notion that the starting point of the slug tail could be considered to be a stagnation point under these flow conditions. However, the reason why void fraction does not influence the application of Bernoulli's equation along the top wall of the pipe is not understood.

Figures 13 and 14 give comparisons between measured and calculated pressure drops over the whole slug when the liquid flow rate was kept at the shedding rate for a Benjamin bubble. Table 2 lists the pressure drops measured in the experiments and calculated from the theory when the liquid flow rate is larger than the shedding rate for a Benjamin bubble. The good agreement between measurements and calculations supports the model presented for a stationary slug.

(C) Slug flow regime

If a slug is created in the pipe, and the flow rate of liquid is decreased, the slug will disappear when the flow is below a minimum value. It is suggested in the theory section that this minimum is equal to the shedding rate of a Benjamin bubble,

$$(V_S)_{\min} = 0.54\frac{\pi D^2}{4}\sqrt{gD}, \qquad [53]$$

if the influence of aeration can be ignored. As seen in figure 5, the critical flows agree exactly with [52], even though the aeration can be large and the void fraction may be >0.3. This result is consistent with what Ruder *et al.* (1989) obtained in studies of a pipeline with i.d. = 9.53 cm. In their experiments, the largest void fraction was >0.3, and the largest Froude number was about 8. This indicates that, under their experimental conditions, aeration does not affect the minimum shedding rate of liquid at the slug tail, i.e the limiting condition for the existence of a slug.

When the flow rate of liquid is increased, the height of the liquid at station 6 and the slug length increase. However, the liquid was not found to fill the whole space of the pipe, even for liquid flows that are three times as large as the shedding rate for a Benjamin bubble in the 2.5 cm pipe. This result supports the notion that there is no upper limit for the existence of a stationary slug in a circular pipe.

Table 1. Pressure drop at the rear of the slug when the shedding rate is the same as for a Benjamin bubble

h_{L1}/D	0.392	0.371	0.360	0.348	0.306	0.282	0.267	0.247	0.230
$u_{L1}/\sqrt{gD}$	1.741	1.809	1.884	2.109	2.321	2.515	2.696	2.804	2.854
$(1-\epsilon_3)$	1.000	0.990	0.964	0.934	0.880	0.834	0.811	0.790	0.768
$(\Delta P_r)_{Th}$	0.147	0.150	0.158	0.168	0.190	0.208	0.224	0.235	0.248
$(\Delta P_r)_{Ex}$	0.156	0.156	0.161	0.175	0.198	0.213	0.235	0.238	0.245

ΔP stands for $\Delta P/\rho_L gD$.

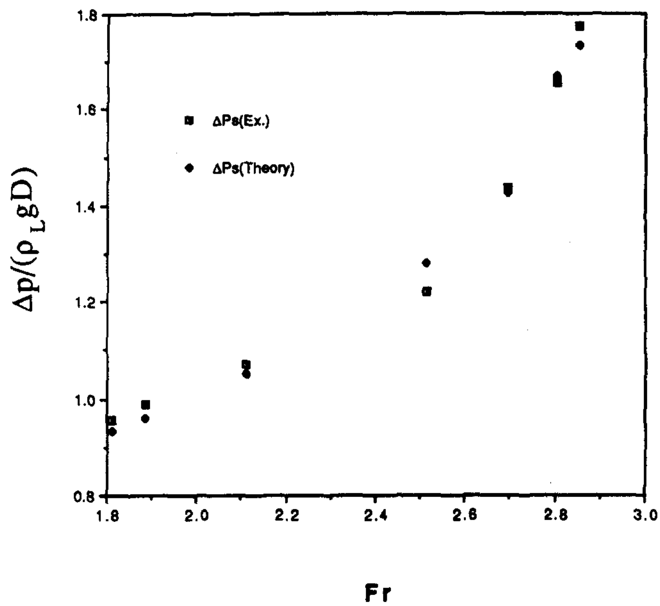


Figure 13. Pressure drop over slugs in the 5.09 cm i.d. pipe.

The analysis of section F showed that, when

$$\frac{h_{L1}}{D} = 0.4456 \quad \text{and} \quad \mathrm{Fr} = \frac{u_{L1}}{\sqrt{gD}} = 1.2579,$$

the pressure drop due to a hydraulic jump, $(\Delta P)_h$, becomes zero. This was suggested as a possible criterion for the transition from slug to plug flow. During the experiments, it was found that when the height of the liquid layer at station 1 is about $0.44D$ and $\mathrm{Fr} \approx 1.26$, no aeration occurs in the front of the slug. The shape of the slug front is very similar to that of the slug tail, and the slug often divides into two consecutive slugs giving an air space between them that looks like a symmetric pocket. With a continued increase in the height of the liquid layer at station 1, there will be several places in the pipe at which the liquid blocks the pipe and several air pockets. The

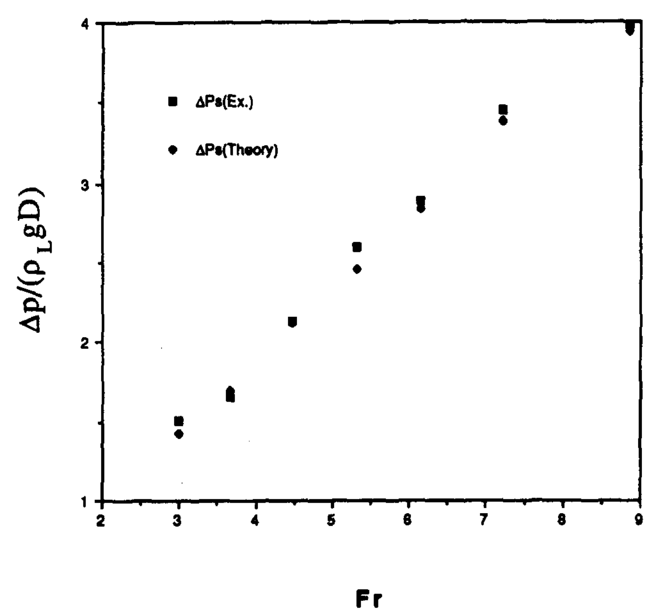


Figure 14. Pressure drop over slugs in the 2.50 cm i.d. pipe.

Table 2. Comparison between the pressure drop measured in the experiments and that calculated from the theory when the shedding rate is larger than for a Benjamin bubble

5.09 cm pipe				2.50 cm pipe			
h_{L1}/D	$u_{L1}/\sqrt{gD}$	$(\Delta P)_{Th}$	$(\Delta P)_{Ex}$	h_{L1}/D	$u_{L1}/\sqrt{gD}$	$(\Delta P)_{Th}$	$(\Delta P)_{Ex}$
0.348	2.041	1.257	1.214	0.238	3.153	1.316	1.432
0.306	2.198	1.318	1.285	0.208	4.872	2.095	2.312
0.348	2.355	1.591	1.593	0.184	6.189	2.814	2.954
0.306	2.512	1.965	1.892	0.151	7.087	3.404	3.483
0.306	2.669	2.154	1.902	0.151	7.984	4.002	4.077
0.282	2.826	2.234	2.286	0.151	8.032	4.061	4.174
0.306	2.932	1.837	1.884	0.129	9.921	5.723	5.714
0.267	2.893	2.191	2.225	0.129	10.97	6.129	6.839
0.247	3.351	2.357	2.381	0.129	11.81	7.032	7.207
0.247	3.770	2.781	2.877	—	—	—	—

ΔP stands for $\Delta P/\rho_L gD$.

flow of liquid becomes very unsteady even though the pressure at the discharge is kept constant. The behavior is cyclical. At first, some long liquid blocks are formed at the upper part of the pipe. These, then, divide into shorter liquid blocks and flow down the pipe, while long liquid blocks form again at the upper part of the pipe. The pressure drop over these liquid blocks is too small to measure, so they cannot be considered as normal steady stationary slugs. It is felt that their appearance represents the transition from slug to plug flow observed for gas/liquid flow in a pipe.

CONCLUSION

Experiments and theory confirm previous results that the front of a stationary slug is a hydraulic jump and show that the liquid height at the rear may be considered to decrease in two stages, a rapidly changing inviscid flow and a slowly changing viscous flow. For a given height at the front of the slug, the first stage of the slug tail is a Benjamin bubble if the volumetric flow rate of liquid is the minimum required for slugs to exist. This conclusion holds even when the slug is highly aerated.

At liquid flows above this minimum the first part of the slug tail does not behave as a Benjamin bubble. It is observed that under these conditions a thin film of air exists at the top of the slug. The results can be explained if the condition that a stagnation point exists at the top of the tail is abandoned. It is argued that this type analysis is consistent with the observation of an air layer.

The observation that a Benjamin bubble exists at the minimum volumetric flow suggests for increasing $Fr = u_{L1}/\sqrt{gD}$ that a lower value of h_{L1}/D is needed for the existence of a stationary slug. The conditions that the energy dissipation and the pressure drop in a hydraulic jump must be positive set lower values of $u_{L1}/\sqrt{gD}$, for all h_{L1}/D, needed for the existence of a hydraulic jump.

Pressure drop measurements over slugs confirm the simple model developed for a stationary slug. It is surprising that the pressure change at the rear of the slug, measured under conditions of severe aeration, agrees with the simple theory that uses a Bernoulli equation at the top wall of the pipe.

The results of this study lend support to theory used to define minimum values of h_{L1}/D required for the existence of slugs in a pipeline flow by Ruder *et al.* (1989). The condition that the pressure change in the hydraulic jump must be positive could explain the transition from a slug to a plug flow regime observed in pipeline flows by Ruder & Hanratty (1990).

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